ALGEBRA.

175. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the conditions that $\dfrac{x}{m+3}+\dfrac{y}{m+1}+\dfrac{z}{m-z}=1$, where m may be a, b or c.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x=(m+3)u$, $y=(m-1)v$, $z=(m-z)w$.

$\therefore$ $u+v+w=1$. $\therefore$ u, v, w are the areal coördinates of a point. Let d, e, f be the sides of the triangle of reference; then

$$\frac{u}{d\alpha}=\frac{v}{e\beta}=\frac{w}{f\gamma}=\frac{1}{2\triangle},$$

where $\triangle$ =area of triangle of reference, and $d\alpha+e\beta+f\gamma=2\triangle$.

$$\therefore\ u=d\alpha/2\triangle,\ \therefore\ x=\frac{(m+3)d\alpha}{2\triangle};$$

$$v=e\beta/2\triangle,\ \therefore\ y=\frac{(m-1)e\beta}{2\triangle};$$

$$w=f\gamma/2\triangle,\ \therefore\ z=\frac{(m-z)f\gamma}{2\triangle},\ \text{or}\ z=\frac{mf\gamma}{2\triangle+f\gamma}.$$

$$\therefore\ \frac{x}{m+3}+\frac{y}{m-1}+\frac{z}{m-z}=\frac{d\alpha+e\beta+f\gamma}{2\triangle}=\frac{2\ \triangle}{2\ \triangle}=1,$$ whatever the value of m.

176. Proposed by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Solve $x^2+y^2+z=a$.....(1), $x+y^2+z^2=b$.....(2), $x^2+y+z^2=c$.....(3).

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x^2+y^2+z^2=s$.

$\therefore$ $z^2-z=s-a$, or $z=\frac{1}{2}\pm\sqrt{(s-a+\frac{1}{4})}=\frac{1}{2}\pm\sqrt{(m-a)}$.

$x^2-x=s-b$, or $x=\frac{1}{2}\pm\sqrt{(s-b+\frac{1}{4})}=\frac{1}{2}\pm\sqrt{(m-b)}$.

$y^2-y=s-c$, or $y=\frac{1}{2}\pm\sqrt{(s-c+\frac{1}{4})}=\frac{1}{2}\pm\sqrt{(m-c)}$.

$\therefore$ $x^2+y^2+z^2=s=\frac{3}{4}\pm[\sqrt{(m-a)}+\sqrt{(m-b)}+\sqrt{(m-c)}]+3m-(a+b+c)$.

$\therefore$ $2m+1-(a+b+c)=\mp[\sqrt{(m-a)}+\sqrt{(m-b)}+\sqrt{(m-c)}]$.

$\therefore$ $2m+D\pm\sqrt{(m-a)}=\mp[\sqrt{(m-b)}+\sqrt{(m-c)}]$.....(1). Squaring (1),

$4m^2+(4D-1)m+D^2+b+c-a=2\{\sqrt{[(m-b)(m-c)]}$

$\mp(2m+D)\sqrt{(m-a)}]$.....(2).